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« Regional Growth and Convergence : Heterogeneous Reaction Versus Interaction in Spatial Econometric Approaches »

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Abstract
This paper presents various approaches dealing with heterogeneous reaction combined with interaction between neighboring units of observation developed in the spatial econometric literature, in the framework of cross-sectional models, and applied to the study of growth and convergence processes. We present the main econometric specifications capturing discrete or continuous spatial heterogeneity: the spatial regimes model and the locally linear, geographically weighted regression (GWR). We then examine how these specifications can be extended to further allow for spatial autocorrelation.

Keywords: spatial econometrics, heterogeneity, spatial autocorrelation
JEL: C21, O47, R11

Résumé
La présence d'effets spatiaux, hétérogénéité et dépendance spatiales, dans les processus de croissance et de convergence régionales a récemment été mis en évidence dans la littérature. Cet article est une introduction aux différentes approches méthodologiques, développées en économétrie spatiale dans le contexte des modèles en coupe transversale et appliquées à l'étude des processus de croissance, dont l'objectif est de traiter le problème de l'hétérogénéité combinée à l'interaction entre unités d'observations voisines. Les spécifications d'économétrie spatiale intégrant l'hétérogénéité spatiale discrète ou continue, en particulier le modèle à régimes spatiaux et le modèle de régression localement linéaire, géographiquement pondéré (GWR) sont d'abord présentées. Deux approches alternatives permettant la prise en compte simultanée de l'hétérogénéité spatiale continue et de la dépendance spatiale sont ensuite développées.

Mots-clés : Econométrie spatiale, hétérogénéité, autocorrélation spatiale
JEL : C21, O47, R11

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"Notwithstanding the general rule that 'everything affects everything else', it is often useful to assess whether the dominant effects are caused by ***reaction*** to external forces or by ***interaction*** between (neighbouring) individuals."[1]

(Cliff and Ord, 1981, p.141)

1. Introduction

Over the last few years, numerous studies have been carried out to analyze economic convergence among countries or regions and recognizing at the same time the need to include spatial effects (Abreu *et al*., 2005; Ertur *et al.*, 2006; Fingleton and López-Bazo, 2006). For example, a large number of contributions analyzing the β-convergence hypothesis impose strong homogeneity assumptions on the cross-economy growth process, since each economy is assumed to have an identical aggregate production function. However, modern growth theory suggests that different economies should be described by distinct production functions. In other words, β-convergence models should account for parameter heterogeneity (Brock and Durlauf, 2001; Durlauf, 2001; Durlauf *et al.*, 2005; Temple, 1999). Evidence of parameter heterogeneity has been found in non-spatial models using different statistical methodologies, such as in Canova (2004), Desdoigts (1999), Durlauf and Johnson (1995) and Durlauf *et al.* (2001). Each of these studies suggests that the assumption of a single linear statistical growth model applying to all countries or regions is incorrect.

Moreover, Ertur *et al*. (2007) argue that in a spatial context, similarities in legal and social institutions, as well as culture and language might create *spatially* local uniformity in economic structures, leading to situations where rates of convergence are similar for

[1] The terms reaction and interaction are emphasised by the authors.

observations located nearby in space. Parameter heterogeneity is then *spatial* in nature and estimating a “global” relationship between growth rate and initial per capita income, which applies in the same way over the whole study area, doesn’t allow capturing the important convergence rate differences that might occur in space.

The instability in space of economic relationships illustrated by this example is called *spatial heterogeneity*. This phenomenon can be observed at several spatial scales: behaviors and economic phenomena are not similar in the center and in the periphery of a city, in an urban region and in a rural region, in the “West” of the enlarged European Union and in the “East”, etc. In an econometric regression, these differences may appear in two ways: with space-varying coefficients and/or space-varying variances. The first case is labeled structural instability of regression parameters, which vary systematically in space. The second case pertains to heteroscedasticity, which is a frequent problem in cross-sections.

Spatial heterogeneity is one of the two spatial effects analyzed by the field of spatial econometrics (Anselin, 1988). This effect operates through the specification of the *reaction* of the variable of interest to explanatory variables or the specification of its variance. The other is spatial autocorrelation, or the coincidence of value similarity and locational similarity. It is aimed at capturing *interaction* between neighboring units of observation. This effect is also highly relevant in growth and convergence analysis. Indeed, as pointed out by Easterly and Levine (2001), there is a tendency for all factors of production to gather together, leading to a geographic concentration of economic activities. As a consequence, any empirical study on growth and convergence should explicitly acknowledge this phenomenon of spatial interdependence between regions or countries. Moreover, as pointed out by Abreu *et al.* (2005), this distinction between spatial heterogeneity and spatial dependence can be related to two different ways of modeling spatial data in growth regressions: models of absolute location and models of relative location. Absolute location refers to the impact of being located at a particular point in space (continent, climate zone) and is usually captured through dummy variables. Relative location refers to the effect of being located closer or further away from other specific countries or regions.

While spatial autocorrelation has been the focus of several literature reviews (Anselin and Bera, 1998; Anselin, 2006 for instance), spatial heterogeneity is much less presented *per se*. In the convergence context, spatial effects have also already been the focus of several literature reviews: Abreu *et al*. (2005), Rey and Janikas (2005) and Fingleton and López-Bazo (2006). However, these studies focus more on the appropriate treatment and interpretation of spatial autocorrelation in convergence models and/or distribution dynamic approaches. Abreu *et al*. (2005) present some models of absolute location but limit their discussion to models for discrete spatial heterogeneity, while several recent studies extend these to models with continuous space-varying coefficients (Bivand and Brunstad, 2005; Eckey *et al.,* 2007; Ertur *et al.,* 2007).

In this context, this chapter is double-aimed. First, we present the main econometric specifications capturing spatial heterogeneity, or models of absolute locations, in the terminology of Abreu *et al*. (2005). Here, we focus on structural instability, as well as on specific forms of heteroscedasticity and we provide examples of applications pertaining to growth econometrics. Secondly, we examine how these specifications can be extended to further allow for spatial autocorrelation in models of heterogeneous reaction. Concerning this second point, it should be noted that spatial autocorrelation and spatial heterogeneity entertain complex links. First, as pointed out by Anselin and Bera (1998) and Abreu *et al*. (2005), there may be observational equivalence between these two effects in a cross-section. Indeed, a cluster of high-growth regions may be the result of spillovers from one region to another or it could be due to similarities in the variables affecting the regions' growth. Secondly, heteroscedasticity and structural instability tests are not reliable in the presence of spatial autocorrelation. For instance, Anselin and Griffith (1988) show that spatial autocorrelation affects the size and power of the White and Breusch-Pagan tests of heteroscedasticity. Anselin (1990a) also provides evidence that the Chow test for structural instability is not reliable in the presence of spatial autocorrelation. Conversely, spatial autocorrelation tests are affected by heteroscedasticity (Anselin, 1990b). Thirdly, spatial autocorrelation is sometimes the result of an unmodelled parameter instability (Brunsdon *et al.*, 1999a). In other words, if space-varying relationships are modelled within a global regression, the error terms may be spatially autocorrelated. We detail some of these issues in the growth and convergence context in this chapter.

Note that heterogeneity can also be modeled using spatial panel data models. However, this alternative approach will not be considered here as a complete survey is provided by Anselin *et al.* (2008). Therefore, we focus here exclusively on the cross-sectional approach. Also, due to space constraints, we do not review distribution dynamics approaches to economic convergence and focus exclusively on spatial econometric modeling issues.[2] Bearing these different elements in mind, this chapter is organized as follows. The next section presents the specifications allowing for discrete heterogeneity, i.e. when different parameters are estimated following spatial regimes. The following sections are devoted to

[2] See Rey and Janikas (2005) and Rey and Le Gallo (2008) for such a review.

continuous heterogeneity models: geographically-weighted regressions (section 3) and their generalizations (section 4). Section 5 concludes and provides some research directions.

2. Discrete spatial heterogeneity

Spatial instability of the parameters necessitates specifications in which the characteristics of each spatial observation are taken into account. Therefore, we could specify a different relationship for each zone *i* of the sample:

$$y_i = x_i^{'}\beta_i + \varepsilon_i \qquad i = 1, \ldots N \tag{1}$$

where y_i represents the observation of the dependent variable for zone *i*; $x_i^{'}$ is the $(1, K)$ vector including the observations for the *K* explanatory variables for zone *i*. It is associated to β_i, a $(K,1)$ vector of parameters to be estimated. Finally, in general, the variance differs with *i*: $\varepsilon_i \sim iid(0, \sigma_i^2)$. Of course, given *N* observations, it is not possible to estimate consistently *NK* parameters and *N* variances: this is the incidental parameter problem. Therefore, a spatial structure for the data must be specified. The spatial variability of the mean of the coefficients of a regression can be *discrete*, if systematic differences between regimes are observed, or it can be *continuous* over the whole area. Note that, similarly to panel data models, a *random* variation could also be specified, under the form of a random coefficients model. As this possibility is not explicitly spatial, it will not be further considered in this chapter.[3]

Consider first the models for discrete spatial heterogeneity, which have been applied extensively to study the club convergence hypothesis in a spatial context. Assume that the area under study is divided into several regimes. If only one variable is under study, a spatial ANOVA can be undertaken in order to investigate whether the mean of this variable is different across the regimes. Spatial versions of ANOVA have also been suggested by Griffith (1992).

[3] See Anselin (1988) for further details on the random coefficients model in a cross-sectional context. See also Brunsdon *et al*. (1999b) for a comparison between random coefficients models and the GWR model, which is considered in section 3 of this chapter.

More generally, in a regression model, consider the case of two regimes, indicated by 1 and 2.[4] It can be written as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \quad (2)$$

where y_1 and y_2 are the $(N_1,1)$ and $(N_2,1)$ vectors of observations for the dependent variable; X_1 and X_2 are the (N_1,K) and (N_2,K) matrices of observations of the explanatory variables; β_1 and β_2 are the unknown vectors of parameters to be estimated. Let $\varepsilon' = \left[\varepsilon_1^{'}\ \varepsilon_2^{'}\right]$ be the vector of error terms. If $E\left[\varepsilon\varepsilon'\right] = \sigma^2 I_N$, the test of spatial homogeneity $\beta_1 = \beta_2$ can be performed with the traditional Chow test. However, more sophisticated error structures can be specified, such as groupwise heteroscedasticity (3) and/or spatial error autocorrelation (4):

$$\Psi = E\left[\varepsilon\varepsilon'\right] = \begin{bmatrix} \sigma_1^2 I_{N_1} & 0 \\ 0 & \sigma_2^2 I_{N_2} \end{bmatrix} \quad (3)$$

$$\text{or} \qquad \varepsilon = \rho W\varepsilon + u \quad u \sim iid(0,\sigma_u^2) \quad (4)$$

where W is a (N,N) spatial weights matrix. Both possibilities can be combined or a different spatial process may be specified for each regime. In each case, maximum likelihood should be carried out and the Chow test must be spatially adjusted (Anselin, 1990a).

This framework has been applied to consider specific forms of parameter heterogeneity in absolute β-convergence regressions, in which case the explanatory variable is the growth rate of per capita income and the explanatory variable is the initial per capita income. Indeed, while absolute β-convergence is frequently rejected for large sample of countries and regions, it is usually accepted for more restricted samples of economies belonging the same geographical area. This observation can be linked to the presence of *convergence clubs*: there is not only one steady state to which all economies converge. From an econometric point of view, one equation must be estimated for each club and decisions must be made as to how the cross-sectional sample should be partitioned.

Some papers just use *a priori* spatial regimes, such as Northern and Southern European regions (Neven and Gouyette, 1995) or regions belonging to cohesion countries and the others (Ramajo *et al.*, 2008). Exploratory spatial data analysis may also prove useful in this task. Indeed, these techniques, by exploiting the specific spatial nature of the data are useful in characterizing the form of spatial heterogeneity by detecting the local concentrations

[4] The generalization to more than two regimes is straightforward.

of similar values, by using Getis-Ord statistics (Ord and Getis, 1995) or LISA statistics (Anselin, 1995). For example, Le Gallo and Ertur (2003) show that the spatial distribution of per capita GDP in Europe before the recent enlargement is characterized by a strong North-South polarization. More recently, Ertur and Koch (2006a) show that this polarization scheme is replaced by a new West-East polarization scheme if the last enlargement of the European Union to Central and Eastern European countries is taken into account. These polarization schemes represent evidence in favor of the existence of at least two spatial regimes in the European regions. This information is then used to estimate β-convergence models with spatial regimes as in equation 2 (Fischer and Stirböck, 2006; Le Gallo and Dall'erba, 2006), possibly associated with groupwise heteroscedasticity, and spatially autocorrelated error terms as in equations 3 and 4 (Ertur *et al*., 2006), or a spatial lag of the form Wy (Dall'erba and Le Gallo, 2008). However, as pointed out by Rey and Janikas (2005), the existing specification search procedures should be extended to be able to distinguish between spatial dependence and spatial heterogeneity while formal specification search strategies for spatial heterogeneity have yet to be suggested.

While non-spatial papers use endogenous detection methods, such as regression trees (Durlauf and Johnson, 1995), it should be emphasized that a technique allowing for an endogenous estimation of regimes together with taking into account of spatial autocorrelation stills needs to be developed (Anselin and Cho, 1998). A first step in this direction is the paper by Basile and Gress (2005) who suggest a semi-parametric spatial autocovariance specification that simultaneously takes into account the problems of non-linearities and spatial dependence. In that purpose, they extend Liu and Stengo's (1999) non-parametric specification by allowing a spatial lag term or a spatial error process.

If no information is available on spatial regimes, or if one thinks that the mean of a variable or that the regression coefficients do not change brutally between regimes, it is preferable to use specifications allowing for continuous spatial variations across the whole study area. The urban literature has frequently used trend surface analysis models and/or the expansion method. In the first case, the coordinates of each location (such as latitude and longitude) are added in the regression model so that the main characteristics of the regression surface, such as simple "North-South" or "East-West" drifts or more complex drifts for higher-order functions can be described (Agterberg, 1984). In the second case, the regression coefficients are deterministic (Casetti, 1972) or stochastic (Anselin, 1988) functions of expansion variables, such as the coordinates of each location. However, the expansion method suffers from two main drawbacks (Fotheringham *et al*., 2000, 2004). First, these techniques only allow capturing trends in relations in space, the complexity of these trends being determined by the complexity of the specified expansion equations. The estimates of the parameters may therefore obscure important local variations to the broad trends represented by the expansion equations. Secondly, the form of the expansion equations must be specified *a priori*. To overcome these problems, Geographically Weighted Regression (GWR) has been developed and applied in several papers focusing on economic convergence.

3. Geographically weighted regression (GWR)

The geographically weighted regression (GWR), or equivalently the locally linear regression method (LWR), has been developed by McMillen (1996) and Brunsdon *et al.* (1996). Most details concerning this method are developed in two books (Fotheringham *et al*., 2000, 2004). GWR is a locally linear, non-parametric estimation method aimed at capturing, for each observation, the spatial variations of the regression coefficients. For that purpose, a different set of parameters is estimated for each observation by using the values of the characteristics taken by the neighboring observations.

Formally, consider again as a point of departure the general formulation (1) where a vector of K unknown parameters must be estimated for each observation i:

$$y_i = x_i^{'}\beta_i + \varepsilon_i = \sum_{k=1}^{K} \beta_{ik} x_{ik} + \varepsilon_i \qquad (5)$$

where $\varepsilon_i \sim iid(0,\sigma^2)$, $i = 1,...,N$. In order to estimate the parameters β_{ik} of model (5), we assume that observations close to location i exert more influence on the estimation of β_{ik} than those located farther away. The idea is then to use a distance-decay weighting scheme that spatially varies with i . Formally, let β_i be the Weighted Least Squares (WLS) estimator of the vector β_i of the K unknown parameters. It is written in matrix form as:

$$\hat{\beta}_i = (X'V_iX)^{-1} X'V_iy \qquad (6)$$

with the same notations as before and $V_i = diag[v_{i1}, v_{i2},..., v_{iN}]$ is a (N,N) diagonal matrix, specific to each location i . The diagonal elements of V_i represent the geographic weighting given to the observations surrounding i, generally specified using a continuous and monotone decreasing function of the distance between location i and all other observations, in other words a kernel function.

This methodology differs from the traditional non-parametric kernel estimation where the weights refer to the attribute space of the explanatory variables (Cleveland *et al.*, 1988). In contrast, GWR uses weights referring to the location in geographical space and therefore allows estimating local rather than global parameters. Different weighting schemes or kernel functions have been suggested in the literature (McMillen, 1996; MacMillen and McDonald, 1997; Fotheringham *et al.*, 2000). One of the most commonly used weighting function is the Gaussian kernel, for a given location i, we have:

$$v_{ij} = \exp(-d_{ij}^2 / h^2) \qquad j = 1, \ldots, N \tag{7}$$

where d_{ij} is the Euclidian distance between locations i and j and h is referred to as the bandwith parameter that can be determined by a cross validation procedure. Another possibility is to use a truncated kernel by setting the weights to zero outside a radius d and to decrease monotonically to zero inside the radius as d_{ij} increases. For example consider a bisquare weighting function written as:

$$v_{ij} = \begin{cases} (1 - d_{ij}^2 / d^2)^2 & \text{if} \quad d_{ij} \le d, \\ 0 & \text{if} \quad d_{ij} > d \end{cases} \tag{8}$$

or even a tri-cube weighting function as suggested by McMillen (1996) and McMillen and McDonald (1997):

$$v_{ij} = \left[1 - \left(\frac{d_{ij}}{d_i} \right)^3 \right]^3 I(d_{ij} < d_i) \tag{9}$$

where d_i is the distance of the *m*th nearest observation to i and $I(\cdot)$ is an indicator function that equals one when the condition is true. The window size, *m*, is the number of nearest neighbors and determines the observations which receive non-zero monotonically decreasing weights, whereas the observation farther away are given zero weights. Again it can be determined by cross-validation.

Note also that a mixed version of GWR has been suggested by Brunsdon *et al.* (1999a) and Mei *et al.* (2004, 2006), in which some coefficients are allowed to vary in space while others remain constant. From an empirical point of view, GWR is useful to identify the nature and patterns of spatial non-stationarity over the studied area. Indeed, the result of a GWR is a set of localized estimations of the parameters, together with localized versions of *t*-statistics and measures of quality of fit. These local measures are associated to specific locations, so that they can be mapped to illustrate the spatial variations of the relationship under study (Mennis, 2006).

We review here some of the most recent contributions related to regional growth and development. Bivand and Brunstad (2005), in their paper focusing on the detection of spatial misspecification in growth models using the R environment, estimate a conditional convergence model including a spatially lagged endogenous variable and spatial regimes for Western Europe over the period 1989-1999. They find support for the role of agricultural subsidies in accounting for variations in regional growth. Higher levels of agricultural support are associated with lower levels of growth, even after some measure of human capital has been introduced. They also consider a GWR specification to essentially ascertain their results by exploring whether any traces of remaining spatial nonstationarity can be found. However, they do not fully interpret their GWR results due to some methodological problems which will be discussed below.

Another attempt to use GWR regressions in the regional growth context has been made by Eckey *et al.* (2007) in a paper focusing on regional convergence in Germany over the period 1995-2002 using disaggregated data on a sample of 180 labor market regions. They estimate a model based on Mankiw *et al.* (1992) allowing all coefficients, especially the rate of convergence, to vary across regions. Each region seems to converge using both absolute and conditional convergence models as the local convergence parameters are all negative. The value of the convergence speed increases from south to north. The half-life period ranges from less than 20 years for some regions in northern Germany to more than 50 years for regions in southern Bavaria.

Finally, let us mention the contribution of Yu (2006) to the regional development literature in his study of the development mechanisms in the Greater Beijing Area using GWR. The analysis reveals two results: first, regional development mechanisms in the Greater Beijing Area, such as Foreign Direct Investment, per capita fixed asset investment and percentage of fixed asset invested in State Owned Enterprises, show significant spatial non-stationarity; and second, development mechanisms have strong local characteristics.

From a methodological point of view, several problems plaguing GWR estimation and inference must be mentioned here. First, concerning statistical inference, in order to know whether the local estimations of parameters are significantly different between them and compared to the OLS estimator, parametric tests have been suggested by Brunsdon *et al.*

(1999a) and Leung *et al*. (2000a). Secondly, LeSage (2004) argues that the presence of aberrant observations due to spatial enclave effects, shifts in regime or outliers can exert undue influence on the GWR estimates. Therefore, he suggests a Bayesian estimation approach that detects these observations and downweights them to lessen their influence on the estimates. Thirdly, Wheeler and Tiefelsdorf (2005) point out that the local regression estimates are potentially collinear even if the underlying exogenous variables in the data generating process are uncorrelated. This collinearity can degrade coefficient precision in GWR and lead to counter-intuitive signs for some regression coefficients. Using Monte-Carlo simulations, Wheeler and Calder (2007) show that Bayesian models with spatially varying coefficients (Gelfand *et al.*, 2003) provide more accurate regression coefficients. Finally, facing the various inference problems encountered by GWR, Páez *et al.* (2002a) place GWR in a different statistical framework, interpreting GWR as a spatial model of error variance heterogeneity, i.e. heteroscedasticity. The variance of the error term is defined as an exponential function of the squared distance between two observations and has then a precise geographical interpretation. While this approach is a special case of the well-known multiplicative heteroscedasticity model developed by Harvey (1976), it nevertheless represents a real breakthrough in the GWR literature and allows the derivation of formal heterogeneity tests.

There still remains an important methodological problem pointed out by Páez *et al.* (2002b) and Pace and LeSage (2004): spatial dependence may not be eliminated even at the optimal bandwidth as it is often assumed in the related literature where it is considered that spatial dependence is mainly due to inadequately modeled spatial heterogeneity. Actually, this methodological problem is related to the complex links between spatial heterogeneity and spatial dependence often underlined and more generally to the *reaction* versus *interaction* debate first pointed out by Cliff and Ord (1981, p.141) in the spatial econometrics literature. Even when heterogeneous reactions are taken into account as in the GWR framework, it could be the case that there are also interactions between units of observation that should be modeled with a spatially dependent covariance structure. Therefore, Brunsdon *et al.* (1998) have proposed to include the spatially lagged endogenous variable in the GWR model and Leung *et al*. (2000b) have suggested a test of spatial autocorrelation of the GWR residuals. Moreover, Páez *et al.* (2002b) formulate a general model of spatial effects that includes as special cases GWR with a spatially lagged endogenous variable (GWR-SL) and GWR with spatially autocorrelated residuals (GWR-SEA). Finally, Pace and LeSage (2004) introduce spatial autoregressive local estimation (SALE) based on a computationally competitive recursive maximum likelihood estimation method.

4. Generalized GWR

We first consider here a straightforward generalization of the model proposed by Páez *et al.* (2002b) where the spatial lags of the explanatory variables are also added in the model:

$$\begin{cases} y = \rho W_1 Y + \tilde{X}\beta + \varepsilon \\ \varepsilon = \lambda W_2 \varepsilon + u \end{cases} \tag{10}$$

where $u \sim N(0,\Omega)$; y is the $(N,1)$ vector of the dependant variable; $\tilde{X} = \begin{bmatrix} \iota & \underline{X} & W_1\underline{X} \end{bmatrix}$ with ι a $(N \times 1)$ vector of ones; $\underline{X}$ a $(N,(K-1))$ matrix of the explanatory variables excluding the constant and $W_1\underline{X}$ its spatial lag; β is the $((2(K-1)+1),1)$ vector of the associated parameters to be estimated; ρ and λ are the spatial autoregressive parameters; W_1 and W_2 are row-standardized spatial weights matrices; Ω is the diagonal covariance matrix of the error term u with elements denoted by ω_{ii}. More precisely, they adopt a specific form for this covariance matrix as follows: $\Omega = \sigma^2 G$ and define its elements as $\omega_{ii} = \sigma^2 g_i(\gamma, z_i)$ and $\omega_{ij} = 0$ for $i \neq j$. Hence the variance of the error term u is a function of a $(p,1)$ vector of known variables z_i, an unobservable parameter vector γ and an unknown constant σ^2. The geographically weighted specification is then obtained by defining a variance model of the exponential form as in Páez *et al.* (2002a):

$$g_{oi}(\gamma_o, a_{oi}) = \exp(\gamma_o d_{oi}^2) \tag{11}$$

which is a special case of the previous formulation with $p=1$ and where the observable variable d_{oi} is the distance between location o and observation i for $i=1,\dots,N$. This particular geographical specification of the error variance is called *locational heterogeneity* by Páez *et al.* (2002a, 2002b). The parameter γ_o is then the so-called kernel bandwidth in the tradition GWR literature. The generalized GWR model can therefore be defined in terms of local parameters as follows:

$$\begin{cases} y = \rho_o W_1 y + \tilde{X}\beta_o + \varepsilon_o \\ \varepsilon_o = \lambda_o W_2 \varepsilon + u_o \end{cases} \Rightarrow \begin{cases} A_o y = \tilde{X}\beta_o + \varepsilon_o \\ B_o \varepsilon_o = u_o \end{cases} \tag{12}$$

where $A_o = I - \rho_o W_1$; $B_o = I - \lambda_0 W_2$ and $u_o \sim N(0, \sigma_o^2 G_o)$. Note that A_o and B_o depend on local parameters ρ_o and λ_o respectively and G_o depends on the local parameter γ_o.

If no spatial lags of the explanatory variables are allowed, i.e. $\tilde{X} = \left[\iota \quad \underline{X}\right]$, it is easily seen that when $\rho_o = \lambda_o = 0$ then $A_o = B_o = I$, and this model reduces to the standard GWR model; when $\lambda_o = 0$ then $B_o = I$, and we obtain a GWR model which includes the spatially lagged endogenous variable (GWR-SL); when $\rho_o = 0$ then $A_o = I$, and we obtain a GWR model with spatially autocorrelated errors (GWR-SEA). Páez *et al.* (2002b) propose to estimate those two generalized GWR models by iterated maximum likelihood. They also derive formal Lagrange multiplier tests against several form of misspecification including a test for omitted endogenous spatial lag, a test for spatial error autocorrelation in GWR models and tests for locational heterogeneity in global models in presence of a spatially lagged endogenous variable or in presence of spatial error autocorrelation. More flexibility is allowed in the specification of the model by using $\tilde{X} = \left[\iota \quad \underline{X} \quad W_1\underline{X}\right]$ which also includes spatial lags of the explanatory variables; the estimation method as well as all of the tests proposed by Páez *et al.* (2002b) may then be straightforwardly generalized to such a model at practically no cost.

An alternative approach to the generalization of the GWR model is proposed by Pace and LeSage (2004): spatial autoregressive local estimation (SALE) allows to simultaneously considering spatial parameter heterogeneity and spatial autocorrelation in an efficient way using recursive spatial maximum likelihood. Their approach is based on the estimation of a sequence of N spatial autoregressions, one for each observation, using a range of sub-sample sizes. Consider the spatial Durbin Model (SDM) where the spatial lags of the explanatory variables are also added in the model:

$$y = \tilde{X}\beta + \rho W y + \varepsilon \tag{13}$$

where the same notations as before and assuming that $\varepsilon \sim N(0, \sigma^2 I_N)$. The concentrated log-likelihood function for the global SDM model is then written as follows, for fixed ρ, omitting the constant term (Pace and Barry, 1997):

$$L(\rho) = \ln\left|I - \rho W\right| - \frac{N}{2}\ln\left[SSE(\rho)\right] \tag{14}$$

where *SSE* denotes the sum of squared residuals. Since the maximum likelihood estimation of the global SDM model relies on least squares estimates and the computation of the log-determinant, a recursive spatial estimation method is conceivable. Pace and LeSage (2004, p. 35) develop such a recursive spatial maximum likelihood approach based on recursive matrix decompositions used to compute log-determinants combined with recursive least squares. More specifically, their approach to compute the log-determinant that appears in (14) relies on the decomposition of $(I-\rho W)$ into two triangular matrices L and U, i.e. $(I-\rho W)=LU$, known as the LU decomposition. It is straightforward to show that:

$$\ln\left|I-\rho W\right|=\ln\left|U\right|=\sum_{j=1}^{N}\ln u_{jj} \tag{15}$$

where u_{jj} is the diagonal element in position (j,j) of the matrix U. Pace and LeSage (2004) underline the recursive nature of the LU decomposition to design a spatial autoregressive local estimation method for the SDM model. Indeed, the log-determinant of the successive sub-matrices are the successive sums of the logarithms of the diagonal elements of the matrix U, so that we have: u_{11} for the first sub-matrix, $\ln u_{11}+\ln u_{22}$ for the second one, and more generally $\sum_{j=1}^{m}\ln u_{jj}$ for the *m*th sub-matrix with $m\le n$.

To implement the estimation procedure for observation i, note that the observations in the sample are first ordered with respect to their distance to observation i. Also, the rows and columns of the weights matrix are consequently reordered. Denote that matrix by W_i. Suppose now that we want to consider sub-samples of size equal to m corresponding to the m-nearest neighbors to observation i. More specifically, the local profile log-likelihood function of Pace and LeSage (2004) is written as follows (omitting the constant term):

$$L_i(\rho_i)=\ln\left|I-\rho_i W_i\right|-\frac{m}{2}\ln\left[SSE(m,\rho_i)\right] \tag{16}$$

It can therefore be rewritten as:

$$L_i(\rho_i)=\sum_{j=1}^{m}\ln u(\rho_i)_{jj}-\frac{m}{2}\ln\left[SSE(m,\rho_i)\right] \quad \text{where} \quad m\le n \tag{17}$$

The recursive method of Pace and Barry (1997) is then used to estimate ρ_i, which may then be interpreted as the *local* spatial autocorrelation parameter. We note that as $m\to N$ these estimates approach the global estimates based on all N observations that would arise from the global SDM model. The procedure is then repeated for all the observations in the sample $i=1,\ldots,N$ yielding a sequence of N spatial autoregressions.

A Bayesian variant of this approach, labeled BSALE has been developed in Ertur *et al.* (2007) in the empirical regional convergence framework and applied to a sample of 138 European regions over the period 1980-1995. On the one hand, regarding heterogeneity as with the standard GWR approach, the proposed locally linear spatial autoregressive model partitions the cross-sectional sample observations by treating each location along with neighboring locations as a sub-sample. This avoids arbitrary decisions regarding how to partition the sample observations, but allows for variation in the parameter estimates across all observations. On the other hand, it is assumed that similarities in legal and social institutions as well as culture and language might give rise to local uniformity in economic structures, leading to similar local schemes for convergence speeds and thus to a concept of *local convergence*. In other words, there should exist spatial clustering in the magnitudes of the β-convergence parameter estimates. However, the locally linear spatial estimation method does not impose *a priori* similar convergence speeds for spatially neighboring observations. Rather, β-convergence parameters for each region in the sample are estimated based on the sub-sample of neighboring regions. Furthermore, Bayesian techniques produce robust estimates with regards to potential outliers and heteroscedasticity of unknown form. A Markov Chain Monte Carlo (MCMC) estimation method is then developed to implement the proposed approach.

The econometric results obtained using different sub-sample sizes show clear evidence that indeed the spatially lagged endogenous variable should be included in the specification. As the sub-sample size increases, they get larger positive modal values for the local spatial autocorrelation coefficients. Individual estimates exhibit local spatial dependence of a sufficiently large magnitude to create bias in standard GWR least-square estimates even for relatively small sub-sample sizes. Estimated local spatial autocorrelation coefficients also present a clear country dependent spatial pattern. Concerning the individual β-convergence parameter estimates, it should be noted that country-level differences are apparent: estimates change abruptly as one move from one country to another. In addition, there is also substantial variation between regions within a country. Samples of draws generated during MCMC sampling are then used to produce confidence intervals. It appears that only 31 regions, mainly located in south-western Europe (Portugal, Spain, some French regions), are converging. All other regions are characterized by non significant estimates. These conclusions are similar for sub-sample sizes varying from roughly one-fourth to three-fourths of the sample size. However, it should be noted that the estimates suffer from sample re-use as in the case of other locally linear non-parametric estimation methods preventing interpretation of the results in a strict statistical sense.

One common criticism that can be made to most of the applications of GWR or SALE presented in the growth and convergence literature is the lack of rigorous theoretical foundations, as the estimated regressions are not derived as reduced forms from structural theoretical models embedding both continuous spatial parameter heterogeneity and spatial interaction.[5] To our knowledge, Ertur and Koch (2007) is the first attempt to develop such a theoretical growth model, which leads to the local SDM model as the relevant econometric reduced form to be estimated. More precisely, their augmented Solow model includes both physical capital externalities as suggested by the Frankel–Arrow–Romer model and spatial externalities in knowledge to model technological interdependence. They suppose that technical progress depends on the stock of physical capital per worker, which is complementary with the stock of knowledge in the home country. It also depends on the stock of knowledge in other countries which affects the technical progress of the home country. The intensity of this spillover effect is assumed to be related to some concept of socio-economic or institutional proximity, which is captured by exogenous geographical proximity. Their model provides, as a reduced form, a conditional convergence equation, which is characterized by complete parameter heterogeneity and which is therefore estimated using SALE on a sample of 91 countries over the period 1960-1995. Their econometric results support their model as all the coefficients have the predicted signs and underline spatially varying convergence speeds across countries as well as varying coefficients for all other explanatory variables and their spatial lags as the saving rates and population growth rates.

Ertur and Koch (2006b) extend this model by including human capital as a production factor following Mankiw *et al.* (1992) and propose to model human capital externalities along the lines of Lucas (1988). Technological interdependence is still modeled in the form of spatial externalities in order to take account of the worldwide diffusion of knowledge across borders. The extended model also yields a spatial autoregressive conditional convergence equation including both spatial autocorrelation and parameter heterogeneity as a reduced form. However, in contrast to Mankiw *et al.* (1992), their results show that the coefficient of human capital is low and not significant when it is used as a simple production factor. Further research is therefore needed to investigate the role played by human capital in growth and convergence processes. In addition, those models having been developed for countries at the international scale, it would be interesting to figure out what modifications are needed to adapt them at the regional scale to help to better understand regional growth and convergence processes.

[5] Until recently, this criticism was also valid for the simpler spatial specifications of convergence models. Some important contributions by Egger and Pfaffermayr (2006), López-Bazo *et al.* (2004), Vayá *et al.* (2004) fill the gap between theoretical and empirical models.

5. Concluding remarks

This chapter aimed at presenting various approaches dealing with heterogeneous reaction eventually combined with interaction between neighboring units of observation developed in the spatial econometric literature, in the framework of cross-sectional models, and applied to growth and convergence processes. Discrete and continuous forms of heterogeneity allowing spatial variations in regressions coefficients have been studied. Geographically Weighted Regressions have been used in the empirical growth and convergence literature to model spatial heterogeneity in regression coefficients and their generalizations taking into account spatial autocorrelation as well as spatial heterogeneity are especially interesting. Further modeling strategies may include newly developed Bayesian models with spatially varying coefficients (Gelfand *et al.*, 2003) and neural networks (Lebreton, 2005). The toolbox of the applied growth researcher is now very diverse and rich. However, most importantly, we believe that, in further research, more efforts should be oriented towards developing, especially at the regional scale, spatial structural theoretical growth models, which would provide the basis of econometric reduced forms that would be estimated using the spatial econometric toolbox.

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